

Phil 183: Critical Reasoning
Week 8: Updating

1. I assign the following probabilities to three mutually exclusive hypotheses (maybe they stand for “Curly did it”, “Moe did it”, “Larry did it”).

H1 -- .25

H2 -- .2

H3 -- .15

And suppose H is the disjunction (H1 or H2 or H3).

- a. If I follow the rules of probability, what is my degree of confidence in H?
 - b. What is my degree of confidence in $\sim H$?
 - c. What is my degree of confidence in (H1 & H2)?
 - d. What is my degree of confidence in $\sim(H1 \text{ or } H2)$?
 - e. What is my degree of confidence in ($\sim H1$ or $\sim H2$)?
2. A theft was committed. It was definitely committed by only one person. It was probably committed by one of the three stooges, Curly, Larry, and Moe. Suppose I have the following degrees of confidence in these mutually exclusive and jointly exhaustive hypotheses:

Curly did it = .25

Larry did it = .2

Moe did it = .15

none of them did it = .4

Suppose I learn that Larry definitely didn't do it.

- a. How confident should I now be in the other hypotheses? Try using the “chocolate bar”, “light cone”, or “tree” method to work this out rather than using Bayes theorem.
- b. How confident should I now be that one of the three stooges did it?

3. You are not sure whether your friend Bob is angry. Generally, Bob is only angry about 2% of the time. But today his face is red. You think the probability that Bob's face would be red if he's angry is about ten times higher than the probability that his face would be red if he's not angry.

After learning that Bob's face is red, how confident should you be that he's angry? Try answering this question by multiplying the prior degree of confidence (in odds form) by the strength factor, rather than a visual method.

4. You are a world-renowned expert on Frida Kahlo's self-portraits. The Detroit Institute of Art has called you in for a consultation to determine whether a portrait they are considering adding to their collection is an authentic Kahlo self-portrait, or merely a fraud created by an impostor. You know that a creative gang has recently flooded the marketplace with well-produced fake artwork, so your degree of confidence that any piece you get called in for is a fake is relatively high, about 20% (and assume that the only possibilities are that it is authentic or that it is fake, and that these possibilities are mutually exclusive). Upon examining the piece, you identify a fairly distinctive brush mark above Kahlo's right cheek in the self-portrait. You know that every one of Kahlo's self-portraits features that distinctive mark, but only about half of fraudulent pieces include it. How confident should you be that the self-portrait is authentic?

5. Ash borers are infecting trees throughout Michigan—about 5% are infected. When a tree is infected, the safest course of action is to cut it down. Your company has an easy test to see whether a particular tree is infected. If a tree is infected, the test will come back positive 90% of the time. Unfortunately, if the tree is not infected, it can still show a positive test 20% of the time. Use this information to answer the following two questions:
- Sally, a homeowner asks you to test one of her trees, and the test comes back negative. When you tell Sally about the test result, she breathes a big sigh of relief and says she's glad to know the tree isn't infected, given the test result. Is Sally drawing the right conclusion? Why/why not?
 - Sally's neighbor Tommy asks you to test one of *his* trees. This time, the test comes back positive. Tommy gets really upset, because he loves the tree and doesn't want to see it cut down. Is Tommy making any mistakes in his probabilistic reasoning?
6. Identify the probability pitfall in each scenario.
- Noelle applied to Southeast Michigan International College (SMIC). The school is very selective and its standards are notoriously secret, and difficult to ascertain. They receive 30,000 applications every year and only admit 30 students per year. She also knows that SMIC acceptance and rejection emails have subject lines with a very particular pattern. 80% of those who are accepted receive an email with a subject line that starts with "Thank you", while 20% of those who are rejected also receive an email with a subject line that starts with "Thank you". One morning, Noelle sees an email from SMIC in her inbox with a subject line starting with "Thank you" and screams for joy, concluding that she probably just got an acceptance email from SMIC.
 - Nastas is on a flight from Detroit to Boston when he feels his seat begin to shake. He tries to think back to his critical reasoning section where this example was discussed and remembers something about $P(E|H)$. He figures out that, indeed, the plane is very likely to shake if it is about to crash land, and begins to scream in terror.

- c. Neil brags to his coworkers about his new trick to drive up sales at the car dealership where they work. He tells his coworkers that he always takes his customers on a test drive route that passes by Michigan Stadium and when they pass it, he tells them, “this car is a real champion.” Inevitably, according to Neil, the customer buys the car upon return to the dealership. One of his coworkers, Naomi, is skeptical, but asks Neil if she can tag along to see the trick in action. Neil agrees and brings Naomi with him on his next test drive with a customer. He uses the same route and makes the remark as usual, but upon return to the dealership, the customer declines to buy the car. Naomi gleefully points out to Neil that the trick doesn’t always work after all. In anger, Neil yells “it doesn’t work when someone is watching, but otherwise, it guarantees a sale!”
- d. Natalya is a Zoology-Computer Science double major, who is preparing to apply for an internship at a hot, new startup called Snapp3Mapp3, which is developing an app where you can send disappearing directions to exclusive parties. It’s steadily growing user base and trendy branding among college students makes it an attractive place to work for ambitious, entrepreneurially-minded young professionals. SnapMap is known for their interest in very-specific college majors, despite the fact that the majority of applicants for their job openings majored exclusively in computer science or marketing. They hire 1% of applicants who majored in Computer Science, 2% of applicants who majored in marketing, but they hire 90% of applicants who majored in zoology, 95% of applicants who majored in astronomy, and 99% of applicants who double-majored in zoology or astronomy along with another major. Natalya’s arch-rival, Ned, hears about this hiring data and decides to taunt Natalya by pointing out that since she majored in Computer Science, she only has a 1% chance of being hired by Snapp3Mapp3.